SCHOLAR Study Guide

Advanced Higher Physics

Appendix: Units, uncertainties and data anaylsis

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SCHOLAR Study Guide Advanced Higher Physics: Appendix: Units, uncertainties and data anaylsis

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Appendix A

Units, prefixes and scientific notation

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Learning objective

By the end of this topic you should be able to:

- understand all the units and prefixes needed for Advanced Higher Physics;
- understand how to correctly use scientific notation and significant figures at this level.

A.1 Physical quantities, symbols and units

The following sections will cover physical quantities, symbols and units used in:

- Rotational motion and astrophysics
- Quanta and waves
- Electromagnetism

A.1.1 Symbols and units used in Rotational motion and astrophysics

Physics Quantity	**Symbol**	**Unit**	**Unit abbreviation**
velocity	v	metre per second	m s^{-1}
displacement	s, x, y	metre	m
acceleration	a	metre per second square	m s^{-2}
time	t	second	s
angular velocity	ω	radian per second	rad s^{-1}
angular displacement	Θ	radian	rad
angular acceleration	α	radian per second square	rad s^{-2}
radius of circle	r	metre	m
torque	T	newton metre	Nm
force	F	newton	N
moment of inertia	I	kilogram metre square	kg m^2
angular momentum	L	kilogram metre square per second	kg m^2 s^{-1}
energy	E	joule	J
universal constant of gravitation	$G\ (= 6.67 \times 10^{-11})$	metre cube per (kilogram second square)	m^3 kg^{-1} s^{-2}
mass of a large object, e.g. planet or star	M	kilogram	kg
mass of a smaller object, e.g. satellite	m	kilogram	kg
gravitational potential	V	joule per kilogram	J kg^{-1}
apparent brightness	b	watt per metre square	W m^{-2}
luminosity	L	watt	W

Physics Quantity	Symbol	Unit	Unit abbreviation
Stefan-Boltzmann constant	$\sigma\ (= 5.67 \times 10^{-8})$	watt per (metre square kelvin quadrupled)	W m^{-2} K^{-4}
temperature	T	kelvin	K
speed of light	$c\ (= 3.0 \times 10^{8})$	metre per second	m s^{-1}

A.1.2 Symbols and units used in Quanta and waves

Physics Quantity	Symbol	Unit	Unit abbreviation
frequency	f	hertz	Hz
planck's constant	$h\ (= 6.63 \times 10^{-34})$	metre square kilogram per second	m^2 kg s^{-1}
wavelength	λ	metre	m
momentum	p	kilogram metre per second	kg m s^{-1}
uncertainty in position	Δ_X	metre	m
uncertainty in momentum, energy, time	$\Delta p, \Delta E, \Delta t$	kilogram metre per second, joule, second	kg m s^{-1}, J, s
charge	q or Q	coulomb	C
magnetic field strength	B	tesla	T
amplitude	A	metre	m
phase difference	φ	radian	rad
fringe widths/spacing	Δx	metre	m
distance between slits and screen	D	metre	m
slit separation/min thickness of lens/width of thin wedge gap	d	metre	m
refractive index	n	no unit	no unit
length of slide (thin wedge)	l	metre	m
Brewster's Angle	i_p	degrees	$^\circ$

A.1.3 Symbols and units used in Electromagnetism

Physics Quantity	Symbol	Unit	Unit abbreviation
permittivity of free space (electric constant)	ε_0 (= 8.85 × 10 $^{-12}$)	coulomb square per newton metre square	C^2 N^{-1} m^{-2}
electrostatic potential	V	joule per coulomb OR volt	J C^{-1} OR V
electric field strength	E	newton per coulomb OR volt per metre	N C^{-1} OR V m^{-1}
current	I	ampere	A
permeability of free space	$\mu_0\ (= 4\pi x 10^{-7})$	metre kilogram per (second square ampere square)	m kg S^{-2} A^{-2}
time constant	t	second	s
resistance	R	ohm	Ω
capacitance	C	farad	F
reactance	X	ohm	Ω
self-induced emf	ε	volt	V
inductance	L	henry	H

Top tip

It can be quicker to use $1/4\pi\varepsilon_0\ \left(= 9 \times 10^9\right)$ in calculations involving the **permittivity of free space (electric constant)**.

Similarly, it can be quicker to use $\mu_0/2\pi\ \left(= 2 \times 10^{-7}\right)$ in calculations involving the **permeability of free space**.

A.2 Prefixes

There are some prefixes that you must know.
These are listed in the following table:

Prefix	Symbol	Equivalent to
pico	p	$\times 10^{-12}$
nano	n	$\times 10^{-9}$
micro	μ	$\times 10^{-6}$
milli	m	$\times 10^{-3}$
kilo	k	$\times 10^{3}$
mega	M	$\times 10^{6}$
giga	G	$\times 10^{9}$

In Advanced Higher Physics you are expected to know and remember the meaning of all of these prefixes. Note that they are from the metric system so are all multiples of 1000 or 1/1000. Hence why centi $\times 10^{-2}$ (eg cm) is not used in the Advanced Higher Physics course.

A.2.1 SI Units

We talk of going 15 miles in a car journey to attend a 5 km race. We use a variety of units in everyday life that can lead to confusion. In science, we have adopted **The International System of Units (SI)**. Sometimes known as the metric system.

SI units all come from seven base units, second (s), metre (m), kilogram (kg), ampere (A), kelvin (K), mole (mol), and candela (cd). The other units derive from these constants.

Symbol	Quantity	Symbol and unit	
a	acceleration	m s^{-2}	metres per second per second
A	activity	Bq	Becquerel
d	distance	m	metre
E	energy	J	Joule
E_h	heat energy	J	Joule
E_k	kinetic energy	J	Joule
E_p	potential energy	J	Joule
E_W	work done	J	Joule
F	force	N	Newton
g	gravitational field strength	N kg^{-1}	Newtons per kilogram
h	height	m	metre
m	mass	kg	kilogram

Symbol	Quantity	Symbol and unit	
p	pressure	Pa	Pascals
P	power	W	Watt
s	displacement	m	metre
t	time	s	second
u	initial velocity	m s^{-1}	metres per second
v	velocity (or final velocity)	m s^{-1}	metres per second
$\bar{v}$	average velocity	m s^{-1}	metres per second
W	weight	N	Newton
Q	charge	C	Coulomb
R	resistance	Ω	Ohm
R_T	total resistance	Ω	Ohm

Symbol	Quantity	Symbol and unit	
T	temperature	K	Kelvin
ΔT	change in temperature	°C	degrees Celsius
V	volume	m^3	metres cubed
V	voltage	V	Volt
V_s	supply voltage	V	Volt
c	specific heat capacity	J kg^{-1} $°C^{-1}$	Joules per kilogram per degree Celsius
I	current	A	Amperes
l	specific latent heat	J kg^{-1}	Joules per kilogram
λ	wavelength	m	metres
ω_R	radiation weighting factor		(no units)
D	absorbed dose	Gy	Gray
f	frequency	Hz	Hertz
H	equivalent dose	Sv	Sievert
$\dot{H}$	equivalent dose rate	Sv s^{-1}	(many possible units)
T	period	s	seconds

A.2.2 Prefixes and Scientific Notation

Scientific Notation is a way of abbreviating, shortening, numbers e.g. 700 becomes 7 $\times$ 10^2 or 29000000 becomes 2.9 $\times$ 10^8.

Scientific Notation makes it simpler to use large and small values.

How do we use it?

700 is the same as 7 $\times$ 10, remember that 100 is 10^2.

So 700 = 7 $\times$ 10^2, they are the same value, just written differently.

When a large number has more than one digit such as 29000000 we use a decimal point after the first digit. The $\times$ 10 power is how many places we have to move the decimal point.

So 29000000 is written 2.9 $\times$ 10^8 because 2.9 $\times$ 10000000 the decimal point has moved eight places.

For numbers less than zero, the movement of the decimal point is labelled as negative. So 0.000005 becomes 5×10^{-6} as it equals $5 \div 1000000$.

Quiz Go online

Q1: Write the following numbers in scientific notation.

a) 64
b) 658423
c) 2345
d) 0.0026
e) 0.000056
f) 0.2304

. . . .

Q2: Write the following numbers in decimals.

a) 1.92×10^{3}
b) 3.051×10^{1}
c) 4.29×10^{7}
d) 1.03×10^{-2}
e) 8.862×10^{-7}
f) 9.512×10^{-5}

Prefixes

To deal with the range of numbers that can appear in physics and allow to make more sense standard prefixes are used to specify multiples and fractions of the units. These are based on Engineering Notation, like Scientific Notation but going up in multiples of 3.

Prefix	Symbol	Multiple	Multiple in full
Giga	G	$\times 10^{9}$	$\times$ 1 000 000 000
Mega	M	$\times 10^{6}$	$\times$ 1 000 000
Kilo	k	$\times 10^{3}$	$\times$ 1 000
Milli	m	$\times 10^{-3}$	$\div$ 1 000
Micro	μ	$\times 10^{-6}$	$\div$ 1 000 000
Nano	n	$\times 10^{-9}$	$\div$ 1 000 000 000

Examples

1. An ultrasound pulse last a time of 852 μs, how many seconds is this?

852 μs = 852 microseconds = 852×10^{-6} s = $852 \div 1\,000\,000$ = 0.000852 seconds.

. . . .

2. A wave has a period of 70 ms. How many seconds is this?

70 ms = 70 milliseconds = 70 $\times$ 10^{-3} s = 70 $\div$ 1 000 = 0.070 seconds.

....

3. A boat travels 54.1 km in an hour. What is the distance the boat travelled in metres?

54.1 km = 54.1 kilometres = 54.1 $\times$ 10^{3} m = 54.1 $\times$ 1 000 = 54 100 metres

....

4. In Back to the Future, the Flux Capacitor required 1.21 GW at 88mph to send Marty back. How much Power is required in Watts?

1.21 GW = 1.21 Giga Watts = 1.21 $\times$ 10^{9} W = 1 210 000 000 Watts

Significant Figures

Often in Maths, they ask you to answer a question to a certain number of significant figures or decimal places.

In Physics, the number of significant figures in the answer is determined by the values in the questions.

Your answer should have the number of significant figures as the smallest number of significant figures given in the question.

Example A greyhound runs round a 515 m field in 12 seconds. Find their average speed?

d = 515 m

t = 12 s

$$\begin{aligned} d &= v \times t \\ 515 &= v \times 12 \\ v &= \frac{515}{12} \\ v &= 42.91666667\ m\ s^{-1} \end{aligned}$$

The 7 in this answer is $\times$ 10^{-8} m every second. Less than the width of a human hair on a run of half a kilometre. It makes no sense to go to that level of precision, especially as our measurements are in metres.

The smallest number of significant figures is 2, for 12 seconds.

The answer becomes $v = 43\ m\ s^{-1}$.

A.3 Scientific notation and significant figures

Scientific notation

When carrying out calculations, you should be able to use scientific notation. This type of notation has been used throughout the topics where necessary, so you should already be familiar with it.

Scientific notation is used when writing very large or very small numbers. When a number is written in scientific notation there is usually one, non-zero number, before the decimal point.

Examples

1. The speed of light is often written as 3.0 x 10^{8} m s^{-1}.

This can be converted into a number in ordinary form by moving the decimal point 8 places to the right, giving 300 000 000 m s^{-1}.

. . . .

2. The capacitance of a capacitor may be 0.000 022 F.

This very small number would often be written as 2.2 x 10^{-5} F. The x 10^{-5} means move the decimal point 5 places to the left.

Top tip

Make sure you know how to enter numbers written in scientific notation into your calculator.

You must give final answers to calculations to an appropriate number of significant figures. In general, this means that your final answer should be given to no more than the least number of significant figures used for the data that was used to obtain the answer.

Significant figures

As a general rule, the final numerical answer that you quote should be to the same number of significant figures as the data given in the question. The above rule is the key point but you might like to note the following points:

1. The answer to a calculation cannot increase the number of significant figures that you can quote.
2. If the data is not all given to the same number of significant figures, identify the least number of significant figures quoted in the data. This least number is the number of significant figures that your answer should be quoted to.
3. When carrying out sequential calculations carry many significant figures as you work through the calculations. At the end of the calculation, round the answer to an appropriate number of significant figures.

Examples

1. The current in a circuit is 6.7 A and the voltage across the circuit is 21 V. Calculate the resistance of the circuit.

Note: Both of these pieces of data are given to two sig. figs. so your answer must also be given to two sig figs.

I = 6.7 A
V = 21 V
R = ?

$$V = I\ R$$
$$21 = 6.7 \times R$$
$$R = 3.1343$$
$$R = 3.1\ \Omega$$

round to 2 sig figs

....

2. A 5.7 kg mass accelerates at 4.36 m s^{-2}.
Calculate the unbalanced force acting on the mass.

Note: The mass is quoted to two sig. figs and the acceleration is quoted to three sig. figs. so the answer should be quoted to two sig figs.

m = 5.7kg
a = 4.36 m s^{-2}
F = ?

$$F = m\ a$$
$$F = 5.7 \times 4.36$$
$$F = 24.852$$
$$F = 25\ N$$

round to 2 sig figs

....

A car accelerates from 0.5037 m s^{-1} to 1.274 m s^{-1} in a time of 4.25 s.
The mass of the car is 0.2607 kg.
Calculate the unbalanced force acting on the car.

Note: The time has the least number of sig figs, three, so the answer should be quoted to three sig figs.

u = 0.5037 m s^{-1}
v = 1.274 m s^{-1}
t = 4.25 s
m = 0.2607 kg

Step 1: calculate a

$$a = \frac{v - u}{t}$$
$$a = \frac{1.274 - 0.5037}{4.25}$$
$$a = 0.181247\ m\ s^{-2}$$

Step 2: calculate F

$$F = m\ a$$
$$F = 0.2607 \times 0.18147$$
$$F = 0.0472511$$
$$F = 0.0473\ N$$

round to 3 sig figs

Finally, it is good practice to check your answers to calculations. Is the answer about what was expected (at least an acceptable order of magnitude)? Do you know a different relationship that can be used to confirm an answer? Does a check on the units confirm that the correct relationship was used (this is called 'dimensional analysis')?

Quiz questions Go online

Q3: A car travels a distance of 12 m in a time of 9.0 s.
The average speed of the car is:

a) 1.3333
b) 1.33
c) 1.3
d) 1.4
e) 1

. . . .

Q4: A mass of 2.26 kg is lifted a height of 1.75 m. The acceleration due to gravity is 9.8 m s^{-2}.
The potential energy gained by the mass is:

a) 38.759 J
b) 38.76 J
c) 38.8 J
d) 39 J
e) 40 J

. . . .

Q5: A trolley of 5.034 kg is moving at a velocity of 4.03 m s^{-1}.
The kinetic energy of the trolley is:

a) 40.878 J
b) 40.88 J
c) 40.9 J
d) 41 J
e) 40 J

A.4 Summary

Summary

You should now be able to:

- have a good knowledge of all the units and prefixes needed for Advanced Higher Physics;
- correctly use scientific notation and significant figures at this level.

Appendix B

Uncertainties

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B.1 Introduction

Uncertainties are covered in some detail at Higher level, but a quick review follows and the main differences at Advanced Higher are that you need to be able to combine uncertainties and work out the uncertainty in the gradient and intercept of a graph. This is covered in detail in the 'Data analysis' section. These skills are highly useful in writing up your project but also could be assessed in the examination.

Whenever a physical quantity is measured, there is always an uncertainty in the measurement - no measurement is ever exact. Uncertainties can never be eliminated but must be reduced as far as possible if experimental results are to be valid.

If an experiment 'does not work' - i.e. the expected result is not obtained - this usually means that the uncertainties in the experimental measurements are very high - so high that the anticipated result may be only obtained by chance. Uncertainties can be reduced by careful experimental design and by experimenters exercising care in the way in which they carry out the experiment and take the measurements. Uncertainties must be taken into account when stating the results of experimental investigation.

Quoting a numerical result of an experiment as (value $\pm$ uncertainty) allows us to check the validity of our experimental method. In addition it enables comparison of the numerical result of one experiment with that of another.

If the result of an experiment to measure a physical quantity of known value (e.g. the speed of light *in vacuo*) leads to a range of values that does not include the accepted value then either the experiment is not valid or, more commonly, the uncertainties have been underestimated. An experiment that leads to a smaller range of uncertainties is more valid than an experiment that has a wider range.

When undertaking experiments you should be prepared to discard or to repeat any measurement that is obviously 'wrong' - i.e. not consistent with the other measurements that you have taken.

There are several causes of uncertainty in experimental measurements and these may be random, scale-reading or systematic.

B.2 Random uncertainties

The effects of random uncertainties are not predictable. For example, when an experimental measurement is repeated several times, the result may not be the same each time. It is likely that some of the readings will be slightly higher than the true value and some will be slightly lower than the true value. Examples could include measurements of time using a stop-watch, measuring an angle using a protractor, measuring length using a measuring tape or ruler.

Random uncertainties are due to factors that cannot be completely eliminated by an experimenter. For example, when taking a measurement of length using a metre stick there may be small variations in the exact positioning of the metre stick from one reading to the next; similarly when reading an analogue meter there may be slight variations in the positions of the experimenter's eyes as readings are taken.

The effects of random uncertainties can be reduced by repeating measurements and finding the mean. The mean value of a number of measurements is the best estimate of the true value of the quantity being measured.

Where a quantity Q is measured n times, the measured value is usually quoted as the mean Q_{mean} of the measurements taken $\pm$ the approximate random uncertainty in the mean. Q_{mean} is the best estimate of the true value and is given by:

$$Q_{mean} = \frac{\Sigma Q_i}{n}$$

The approximate random uncertainty in the mean is given by:

$$approximate\ random\ uncertainty\ = \frac{Q\ maximum\ -\ Q\ minimum}{n}$$

Notes:

1. A random uncertainty can only be calculated from measured data that you would expect to be the same value.
2. A random uncertainty must not be found in calculated values.
3. The above relationship is an approximation; it is not statistically rigorous, but it is sufficiently accurate at this level when at least 5 readings have been taken.

Example A student uses a computer program to measure their reaction time. The following values are obtained for the reaction time of the student.

Attempt number	**1**	**2**	**3**	**4**	**5**
Reaction time /s	0.273	0.253	0.268	0.273	0.238

a) Calculate the mean reaction time of the student.

b) Calculate the approximate random uncertainty in the mean.

a)

$$\begin{aligned} mean =& \frac{total\ of\ values}{number\ of\ values} \\ mean =& \frac{(0.273 + 0.253 + 0.268 + 0.273 + 0.238)}{5} \\ mean =& \frac{1.305}{5} \\ mean =& 0.261 \text{ s} \end{aligned}$$

b)

$$\begin{aligned} random\ uncertainty =& \frac{(max\ value - min\ value)}{number\ of\ values} \\ random\ uncertainty =& \frac{(0.273 - 0.238)}{5} \\ random\ uncertainty =& 0.007 \text{ s} \end{aligned}$$

Interpretation of these calculations

These are often written as:

best estimate = mean value $\pm$ uncertainty

best estimate of reaction time = 0.261 s $\pm$ 0.007 s

This means that if the reaction time was measured again it is likely, not guaranteed, that the value would be with the range of 0.261 s plus or minus 0.007 s.

$\Rightarrow$ Likely that measured value of time would lie between 0.254 s and 0.268 s.

Increasing the reliability

In order to increase the reliability of a measurement, increase the number of times that the quantity is measured. It is likely that the random uncertainty will decrease.
In the above example this would mean that instead of finding the mean reaction time based on 5 attempts, repeat the measurement so that the calculation is based on 10 attempts.

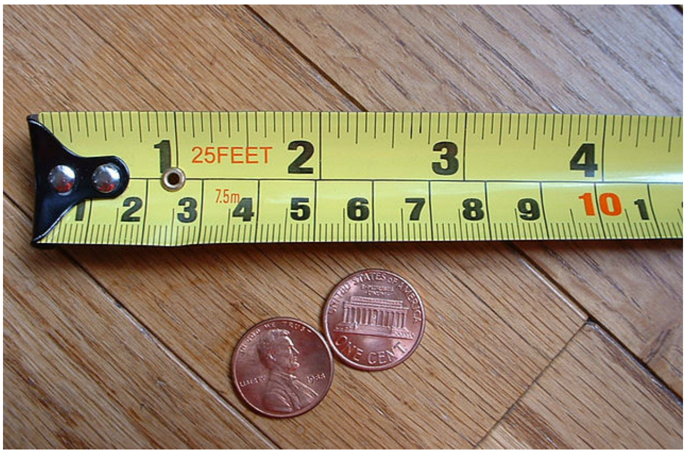
If you repeat a measurement 5 times and you measure exactly the same value on each occasion then the random uncertainty will be zero. Making further repeated measurements is unnecessary as this will not reduce the random uncertainty so it will not increase the reliability.

B.3 Scale-reading uncertainties

A scale reading uncertainty is a measure of how well an instrument scale can be read. This type of uncertainty is generally random and is due to the finite divisions on the scales of measuring instruments. For example, the probable uncertainty in a measurement of length, using a metre stick graduated in 1 mm divisions, is 0.5 mm. If more precision is needed then a different measuring instrument (e.g. a metal ruler or a micrometer) or a different technique must be used.

For instruments with analogue scales, the scale-reading uncertainty is usually taken as $\pm$ half of the smallest scale division. In some cases, it may be possible to make reliable estimates of smaller fractions of scale divisions.

For instruments with digital scales the reading uncertainty is 1 in the last (least significant) digit.

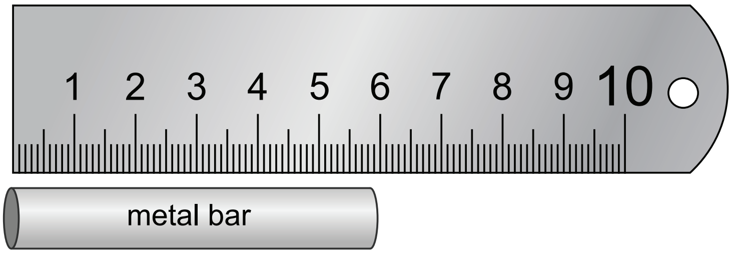


Examples

1. Analogue scale

This approach is used for rulers, metre sticks, liquid in glass thermometer and meters which have a pointer.

The length of metal is measured with the ruler shown below.



Length 6 cm
Scale reading uncertainty = half of one scale division = 0.5 cm

Often expressed as 6.0 cm $\pm$ 0.5 cm

This means that the best estimate of the length is 6.0 cm and it would be expected that the "true" length would be between 5.5 cm and 6.5 cm.

. . . .

2. Digital display

This approach is used whenever a seven segment digital display is present.

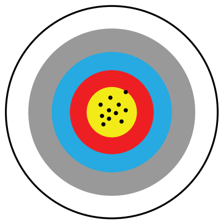
The following image shows a digital ammeter.

Current = 12.9 A
Scale reading uncertainty = one in smallest scale division = 0.1 A

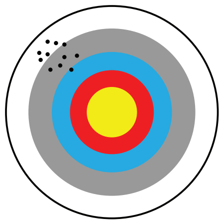
Often expressed as 12.9 A $\pm$ 0.1 A

This means that the best estimate of the current is 12.9 A and it would be expected that the "true" current would be between 12.8 A and 13.0 A

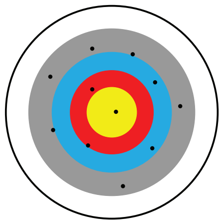
B.4 Systematic uncertainties



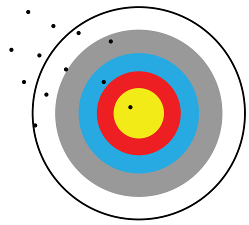
small systematic /
small random



large systematic /
small random



small systematic /
large random



large systematic /
large random

Systematic uncertainties have consistent effects on the quantities being measured.

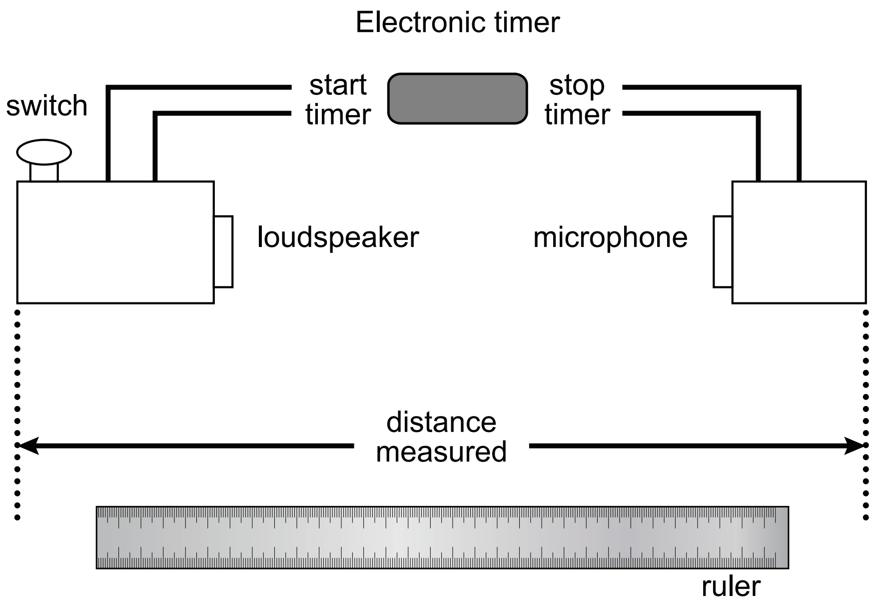
Systematic uncertainties often arise due to experimental design or issues with the equipment.

The following example shows a ruler being used to measure the length of a metal bar.

At first sight the length of the metal bar is 8 cm.
However, on closer inspection the actual length is only 7 cm as the ruler starts at 1 cm rather than 0 cm.
This ruler could easily cause all measured values to be too long by 1 cm. This would be a systematic uncertainty.

This systematic uncertainty could have been noticed by the experimenter and corrected but often the presence of a systematic uncertainty is not detected until data is analysed.

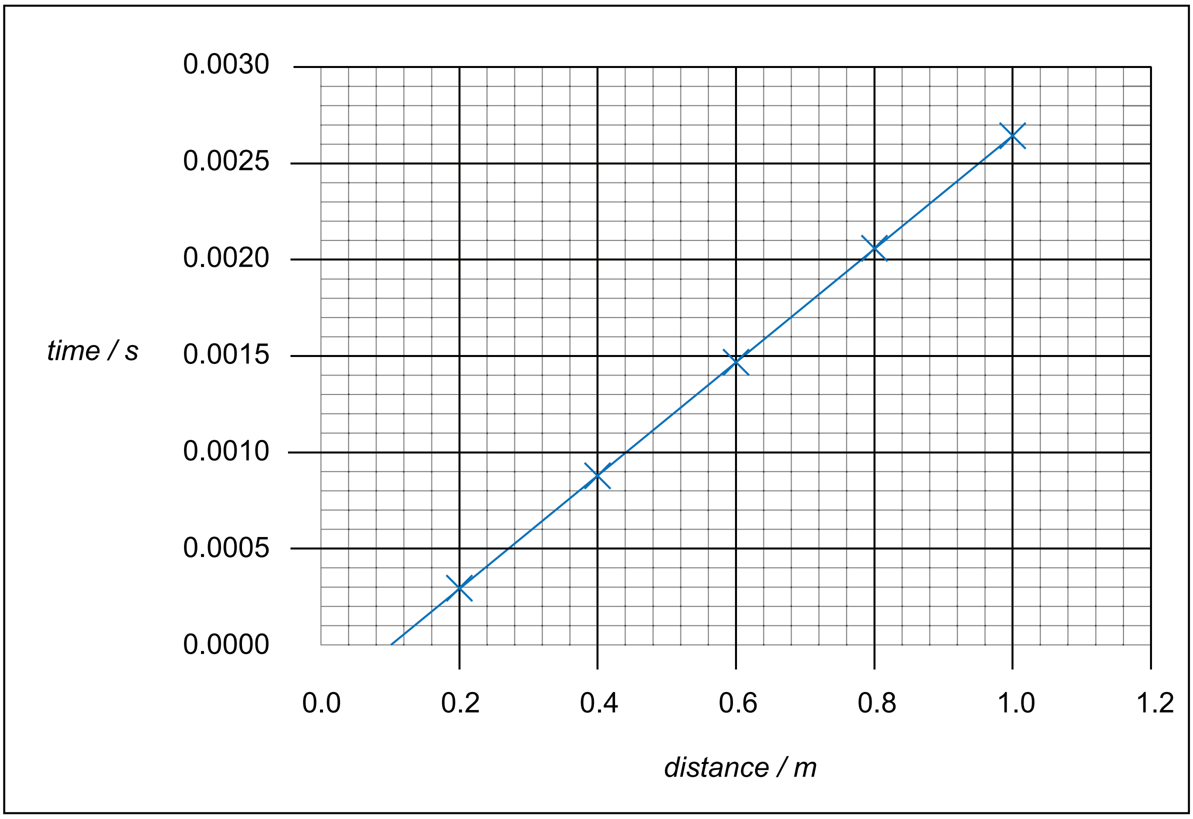
Example A student is investigating how the distance between a loudspeaker and microphone affects the time it takes a pulse of sound to travel from the loudspeaker to the microphone. The equipment used is shown below.



When the switch is pressed the loudspeaker produces a sound and the timer starts. When the sound reaches the microphone the timer is stopped.

The distance shown is measured with a ruler. The distance is altered by moving the microphone to a greater distance from the loudspeaker and further measurements are taken.

The results obtained are displayed on the following graph.



The expected graph is a straight line through the origin.
Here a straight line is obtained but it does not go through the origin.
This shows that there is a systematic uncertainty in the investigation.

The line is too far to the right so **all** of the distance measurements are too big by the same value. There is a systematic uncertainty of 0.1 m. This value is found by finding the intercept on the distance axis.

What has caused this systematic uncertainty?

Look at the labelled diagram and notice that the distance is between the extreme edges of the loudspeaker and the microphone.
The sound will be made inside the loudspeaker box and the microphone will be inside the microphone box. This means that the sound does not have to travel this distance and all the distances measured are too big by 0.1 m.

Further thoughts on this investigation

1. The gradient of this graph can lead to an estimate of the speed of sound.

$$gradient = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$
$$gradient = \frac{\Delta time}{\Delta distance}$$
$$gradient = \frac{(0.0015 - 0)}{(0.6 - 0.1)}$$
$$gradient = 3 \times 10^{-3}$$

   since

$$speed = \frac{\Delta distance}{\Delta time}$$

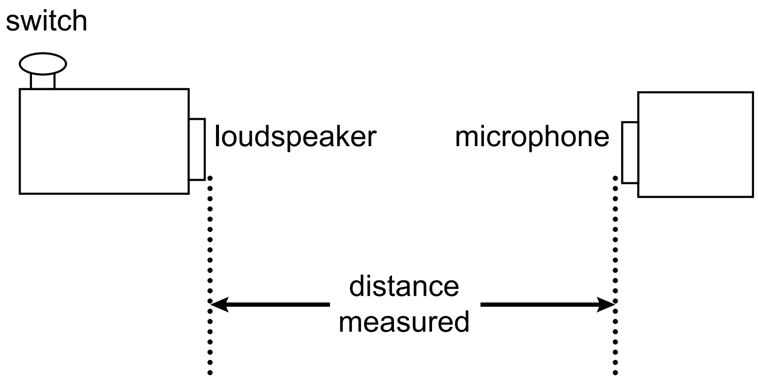
   and here

$$gradient = \frac{\Delta time}{\Delta distance}$$

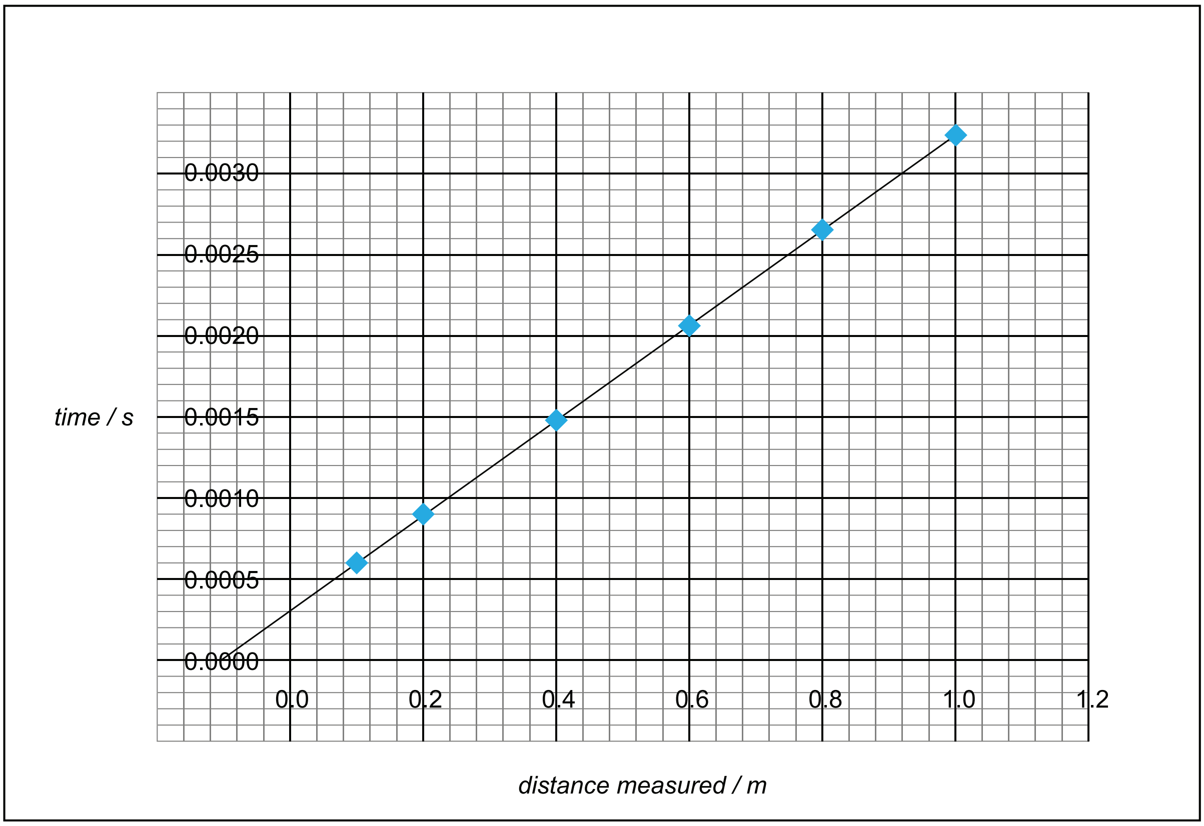
   hence

$$speed = \frac{1}{gradient}$$
$$speed = \frac{1}{3 \times 10^{-3}}$$
$$Speed\ of\ sound\ = 333\ m\ s^{-1}$$

2. It may be suggested that the systematic uncertainty could be removed by measuring the distance between the inside edges of the loudspeaker and microphone as shown in the diagram below.



This would result in the following graph.



Using this approach, a straight line is obtained but again does not pass through the origin indicating the presence of a systematic uncertainty. The line is too far to the left.
The distance measured is too short and the underestimate is always 0.1 m. This value is found from the intercept on the distance axis. This means that all the distance measurements are too small by 0.1 m.

It is impossible to remove the systematic uncertainty unless the actual positions of where the sound is produced and where the sound is detected are known. This cannot be done if the components are mounted inside "boxes".

The gradient of this graph would again give an estimate of the speed of sound.

Identifying systematic effects is often an important part of the evaluation of an experiment.

B.5 Calibration uncertainties

A Calibration uncertainty is a manufacturer's claim for the accuracy of a instrument compared with an approved standard. It is usually found in the instructions that are supplied with the instrument. Calibration uncertainties are often systematic in nature. Calibration uncertainties may be predictable or unpredictable. For example the drift of the time base of an oscilloscope due to temperature changes may not be predictable but it is likely to have a consistent effect on experimental results. Other examples of calibration uncertainties are a clock running consistently fast or consistently slow, an ammeter reading 5% higher than the true reading and a balanced incorrectly zeroed at the start of an experiment reading consistently too high or too low.

Appendix C

Data analysis

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C.4 Centroid or parallelogram method 34
C.5 Accuracy and precision 36

C.1 Calculating and stating uncertainties

Single measurements may be quoted as ± measurement absolute uncertainty, for example 53.20 ± 0.05 cm. When measured quantities are combined (e.g. when the quantities are multiplied, divided or raised to a power) to obtain the final result of an experiment it is often more useful to quote measurement ± percentage uncertainty, where

$$percentage\ uncertainty = \frac{actual\ uncertainty}{measurement} \times 100$$

In an experiment where more than one physical quantity has been measured, the largest percentage uncertainty in any individual quantity is often a good estimate of the percentage uncertainty in the final numerical result of the experiment.

When comparing the uncertainty in two or more measured values it is necessary to compare percentage uncertainties not absolute uncertainties.

In an investigation the distance travelled and the time taken are measured and the results are expressed in the form.

Best estimate ± absolute uncertainty

distance,d	=	125 mm ± 0.5 mm (metre stick, analogue device)
time, t	=	5. 2 s ± 0.1 s (stop watch, digital device)

$$\begin{aligned}
\%uncert\ in\ d =& \frac{absoluteuncert\ in\ d}{measurement\ of\ d} \times 100 \\
\%uncert\ in\ d =& \frac{0.5}{125} \times 100 \\
\%uncert\ in\ d =& 0.4\% \\
\%uncert\ in\ t =& \frac{absolute\ uncert\ in\ t}{measurement\ of\ t} \times 100 \\
\%uncert\ in\ t =& \frac{0.1}{5.2} \times 100 \\
\%uncert\ in\ t =& 2\%
\end{aligned}$$

In order to compare the precision of these two measurements the percentage uncertainty in each measurement must be calculated.

Comparing these two percentage uncertainties it can be seen that the percentage uncertainty in time is much greater than the percentage uncertainty in the distance.

Finding the uncertainty in a calculated value

The uncertainty in a calculated value can be estimated by comparing the percentage uncertainties in the measured values. At Higher level, normally one percentage uncertainty was three or more times larger than all the others and as a result this largest percentage uncertainty was a good estimate of the uncertainty in the calculated value. This may again be the case at Advanced Higher level.

Evaluating an experimental method

In order to improve the precision of an experiment it is necessary to find the measurement with the largest percentage uncertainty and consider how this percentage uncertainty could be reduced. Using the figures given above for distance and time the percentage uncertainty in time is greatest therefore an improvement method of measuring the time is required. Using two light gates connected to an electronic timer would enable the time to be measured with a smaller scale reading uncertainty. This would improve the precision in the measurement of time and hence in average speed.

Example Using the measured values of distance and time given, calculate the average speed of the moving object. In order to carry this out the percentage uncertainties in distance and time must be know.

distance,d	=	125 mm $\pm$ 0.4%
time, t	=	5. 2 s $\pm$ 2%

$$averagespeed = \frac{distance\ gone}{time\ taken}$$

$$averagespeed = \frac{125}{5.2}$$

$$averagespeed = 24\text{mm s}^{-1}$$

The percentage uncertainty in the average speed will be 2%. The percentage uncertainty in t is more than three time the percentage uncertainty in d.

$$averagespeed = 24\text{mms}^{-1} \pm\ 2\%$$

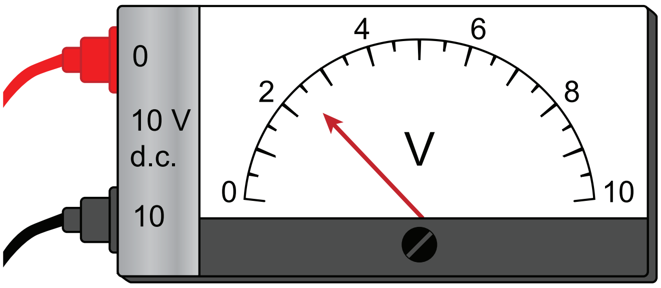
When measured quantities are combined it is usual to ignore any percentage uncertainty that is not significant.

A percentage uncertainty in an individual measured quantity can be regarded as insignificant if it is less than one third of any other percentage uncertainty.

Absolute uncertainty should always be rounded to one significant figure.

Quiz Go online

Q1: State the scale reading uncertainty in the following voltmeter reading.



a) $\pm$ 0.25 V
b) $\pm$ 0.5 V
c) $\pm$ 1.0 V
d) $\pm$ 2.0 V
e) $\pm$ 5.5 V

. . . .

Q2: A student carries out an investigation to measure the time taken for ten complete swings of a pendulum.
The following values are obtained for the time for ten complete swings.

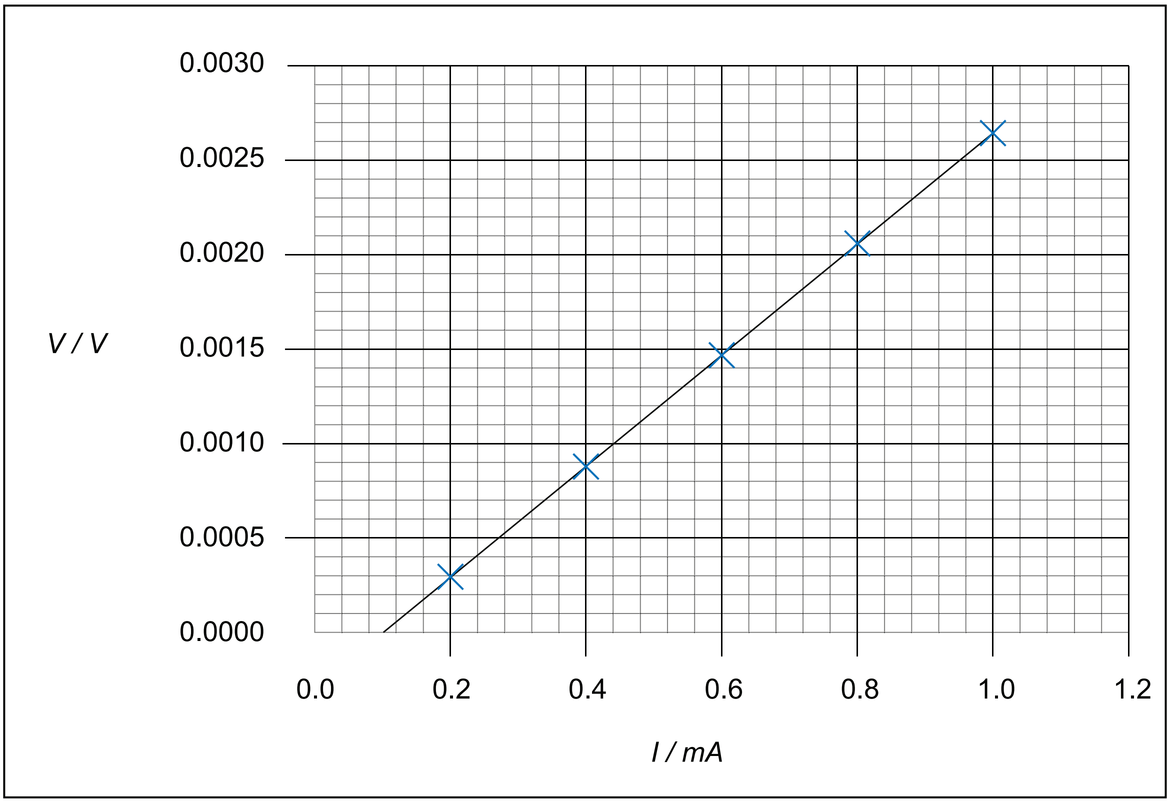
3.1 s	**3.8 s**	**3.3 s**	**4.1 s**	**3.4 s**

What is the random uncertainty in the time for ten complete swings?

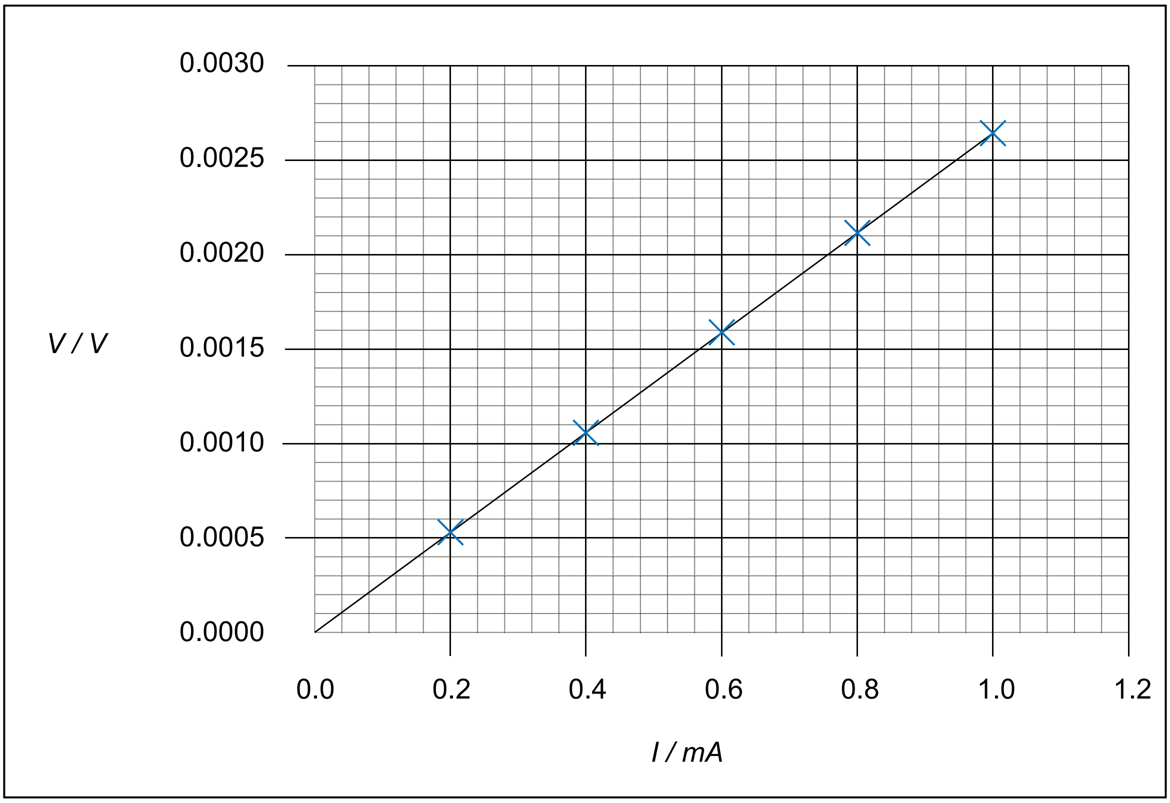
a) $\pm$ 0.01 s
b) $\pm$ 0.02 s
c) $\pm$ 0.1 s
d) $\pm$ 0.2 s
e) $\pm$ 1.0 s

. . . .

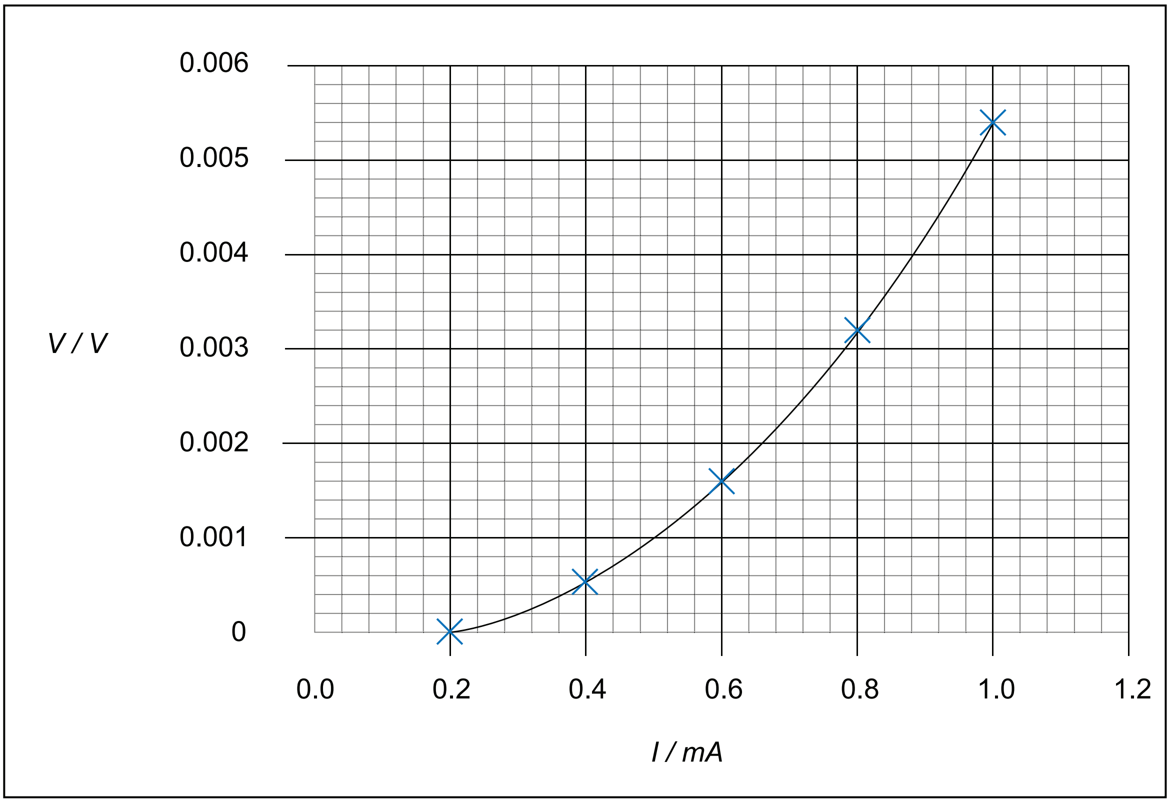
Q3: A student carries out three investigations into the variation of voltage and current. The results obtained are shown in the Graphs A, B and C.



Graph A



Graph B



Graph C

Which of the following statements is/are true?

- I Graph A shows a systematic uncertainty
- II Graph B shows a proportional relationship
- III Graph C shows a systematic uncertainty

a) I only
b) II only
c) I and II only
d) I and III only
e) I, II and III

. . . .

Q4: In an experiment the following measurements and uncertainties are recorded.

Temperature rise	=	10° C $\pm$ 1°C
Heater current	=	5.0 A $\pm$ 0.2 A
Heater voltage	=	12.0 V $\pm$ 0.5 V
Time	=	100 s $\pm$ 2 s
Mass of liquid	=	1.000 kg $\pm$ 0.005 kg

The measurement which has the largest percentage uncertainty is the:

a) Temperature rise
b) Heater current
c) Heater voltage
d) Time
e) Mass of liquid

. . . .

Q5: In an investigation the acceleration of a trolley down a slope is found to be 2.5 m s^{-2} $\pm$ 4%.

The absolute uncertainty in this value of acceleration is:

a) $\pm$ 0.04 m s^{-2}
b) $\pm$ 0.1 m s^{-2}
c) $\pm$ 0.4 m s^{-2}
d) $\pm$ 1.0 m s^{-2}
e) $\pm$ 4.0 m s^{-2}

. . . .

Q6: In an investigation the voltage across a resistor is measured as 20 V $\pm$ 2 V and the current through it is 5.0 A $\pm$ 0.1 A.

The percentage uncertainty in the power is:

a) 0.1%
b) 2%
c) 3%
d) 10%
e) 12%

. . . .

Q7: Specific heat capacity can be found from the experimental results given below.

Which one of the following measurements creates most uncertainty in the calculated value of the specific heat capacity?

a) Power = 2000 $\pm$ 10 W
b) Time = 300 $\pm$ 1 s
c) Mass = 5.0 $\pm$ 0.2 kg
d) Final temperature = 50 $\pm$ 0.5°C
e) Change in temperature = 30 $\pm$ 1°C

. . . .

Q8: The light coming from a spectral lamp is investigated and the following data is obtained.

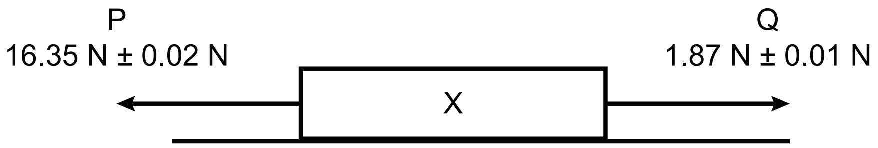
$$\lambda = 450 \text{ nm} \pm 10\%$$
$$f = 6.7 \times 10^{14} \text{Hz} \pm 2\%$$

This data is used to estimate the speed of light. The absolute uncertainty in this estimate of the speed of light is:

a) $\pm$ 2.0 m s^{-1}
b) $\pm$ 10 m s^{-1}
c) $\pm$ 6.0 x 106 m s^{-1}
d) $\pm$ 3.0 x 10^7 m s^{-1}
e) $\pm$ 3.6 x 10^7 m s^{-1}

. . . .

Q9: Two forces P and Q act on an object X as shown.



The value of the unbalanced force acting on the object X and the percentage uncertainty in this value, expressed in the form value $\pm$ absolute uncertainty is:

a) 14.48 N $\pm$ 0.03N
b) 14.48 N $\pm$ 0.08N
c) 14.48 N $\pm$ 0.5N
d) 18.22 N $\pm$ 0.03 N
e) 18.22 N $\pm$ 0.08N

. . . .

Q10: A student measures their reaction time using the digital stop watch on a computer.

The following measurements of their reaction time are displayed on the computer's digital stop watch.

0.29 s	**0.25 s**	**0.22 s**	**0.26 s**	**0.24 s**

When evaluating this set of measurements the student makes the following statements.

- I Increasing the number of attempts from 5 to10 would make the mean value more reliable.
- II The scale reading uncertainty in this set of measurements is $\pm$ 0.01 s.
- III You can tell by reviewing the measurements that there is no systematic uncertainty present.

Which of the above statements is/are correct?

a) I only
b) II only
c) III only
d) I and II only
e) I, II and III

C.2 Combining uncertainties

Uncertainties in combinations of quantities are calculated as follows.

Addition and subtraction

When two quantities P with absolute uncertainty ΔP, and Q with absolute uncertainty ΔQ, are added or subtracted to give a further quantity S, the absolute uncertainty ΔS in S, is given by

$$\Delta S = \sqrt{\Delta P^2 + \Delta Q^2}$$

Multiplication and division

When two quantities P with absolute uncertainty ΔP, and Q with absolute uncertainty ΔQ, are multiplied or divided to give a further quantity S, the percentage uncertainty in S is given by

$$\% \, uncertainty \, in \, S = \sqrt{(\% \, uncertainty \, in \, P)^2 + (\% \, uncertainty \, in \, Q)^2}$$
$$\frac{\Delta S}{S} \times 100 = \sqrt{\left(\frac{\Delta P}{P} \times 100\right)^2 + \left(\frac{\Delta Q}{Q} \times 100\right)^2}$$

Powers

When a quantity P is raised to a power n to give a further quantity Q, then

$$\% \, uncertainty \, in \, Q = n \times \% \, uncertainty \, in \, P$$

When measured quantities are combined it is usual to ignore any percentage uncertainty that is not significant; a percentage uncertainty in an individual measured quantity can be regarded as insignificant if it is less than one third of any other percentage uncertainty.

Example

Calculate the kinetic energy and corresponding absolute uncertainty of a car moving at (25.0 $\pm$ 0.1)ms^{-1} with a mass of (2200 $\pm$ 10)kg.

First you'd find the kinetic energy:

$$E_k = \frac{1}{2}mv^2$$
$$E_k = 0.5 \times 2200 \times 25.0^2$$
$$E_k = 687,500 \text{ J} = 690,000 \text{ J to the correct two significant figures.}$$

Next you should work out the percentage uncertainties in each of the quantities:

$$\%\ uncertainty\ in\ mass = 10/2200 \times 100 = 0.45\%$$
$$\%\ uncertainty\ in\ velocity = 0.1/25 \times 100 = 0.40\%$$

which due to being v^2 is doubled to get 0.80% (see powers equation above)

Next step is to combine the uncertainties using the multiplication rule:

$$\%\ uncertainty\ in\ kinetic\ energy = \sqrt{(0.45^2 + 0.8^2)} = 0.92\%$$

Finally work out the absolute uncertainty which is 0.92% of 690kJ = 6000J

(Remember absolute uncertainties are always quoted to one significant figure.)

Your answer is best expressed as:

Kinetic Energy = (690 $\pm$ 6)kJ

C.3 Uncertainties and graphs

When graphing quantities that include uncertainties, note the following points.

- Each individual point on the graph should include error bars, on one or both of the quantities being plotted, as appropriate.
- The error bars on each point could indicate either the absolute uncertainty or the percentage uncertainty in the quantities being plotted, as appropriate.
- The error bars are used to draw the best straight line or the best fit curve, as appropriate.

C.4 Centroid or parallelogram method

The following method (known as the 'centroid' method) can be used to estimate the uncertainty in the gradient and the uncertainty in the y-intercept of a straight-line graph.

1. Plot the points and error bars.
2. Calculate the centroid of the points. The x co-ordinate of the centroid is the mean of the values plotted on the x-axis and the y co-ordinate of the centroid is the mean of the values plotted on the y-axis.
3. Draw the best fitting straight line through the centroid.
4. Construct a parallelogram by drawing lines, which are parallel to this line, and which pass through the points furthest above and furthest below this line.
5. Calculate the gradients, m_1 and m_2, of the diagonals of the parallelogram. (These two diagonals represent the greatest and least values that the gradient could have.)
6. Calculate the uncertainty in the gradient, Δm using
   $\Delta m = \frac{m_1 - m_2}{2\sqrt{(n-2)}}$
   where n is the number of points (not including the centroid) plotted on the graph.
7. Read off the intercepts, c_1 and c_2, on the *y*-axis, of the diagonals of the parallelogram.
8. Calculate the uncertainty in the intercept, Δc using
   $\Delta c = \frac{c_1 - c_2}{2\sqrt{(n-2)}}$
   Again, n is the number of points (not including the centroid) plotted on the graph.

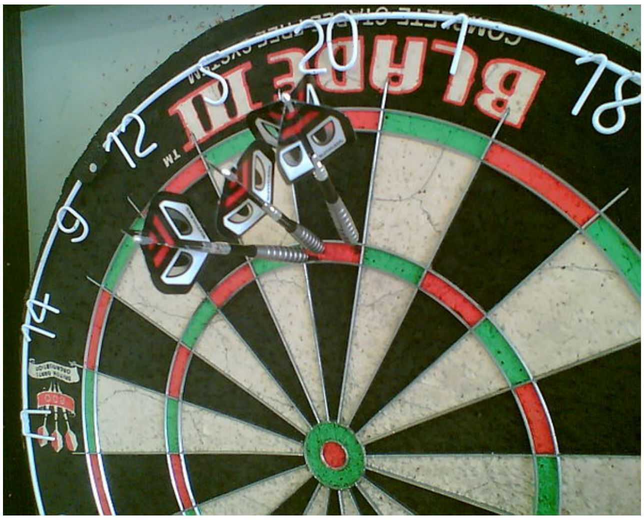
There are various methods possible using computers to calculate the error in the gradient and the intercept. One option is to use one of the functions available in graph drawing software e.g. LINEST and Trendline functions in Excel. Use the Help feature on Excel to find out more or ask your teacher for guidance.

LINEST in exel - (Takes you through the steps to use LINEST)

http://www.colby.edu/chemistry/PChem/notes/linest.pdf

(*last accessed in October 2019*)

C.5 Accuracy and precision

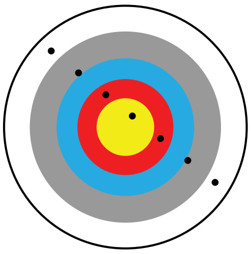


"Dart180" by Sebastian Kamper is licensed under CC BY 3.0

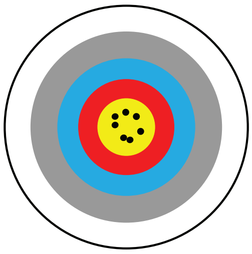
The accuracy of a measurement compares how close the measurement is to the 'true' or accepted value. The darts player in the picture above is clearly accurate! The precision of a measurement gives an indication of the uncertainty in the measurement, the lower the uncertainty the more precise the reading. The darts player is also very precise and has achieved the maximum 180 with three darts.

Accurate, precise, both or neither?

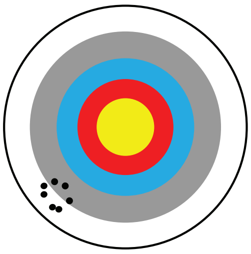
For the following three pictures, would you say they are accurate, precise, both or neither?



(a)

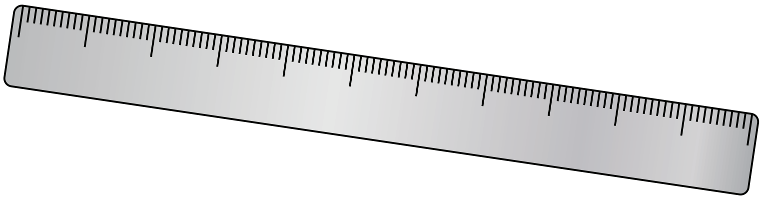


(b)



(c)

Example



If we measure the length of a metre stick and get values of 1.05m, 1.03m, 1.04m, 1.05m would you say these results were precise and/or accurate?

The following link will help you to understand the Accuracy and Precision, Systematic Error and Random Uncertainty

https://www.youtube.com/watch?v=icWY7nICrfo

(*last accessed in October 2019*)

Hints for activities

Appendix A: Units, prefixes and scientific notation

Quiz questions

Hint 1: Data is quoted to 2 sig figs so answer must be quoted to 2 sig figs.

Hint 2: The acceleration due to gravity is quoted to only 2 sig figs so the answer must be given to 2 sig figs.

Hint 3: The mass of the trolley is given to 4 sig figs and the velocity is given to 3 sig figs.

Answers to questions and activities

Appendix A: Units, prefixes and scientific notation

Quiz (page 9)

Q1:

a) 6.4×10^{1}
b) 6.58423×10^{5}
c) 2.345×10^{3}
d) 2.6×10^{-3}
e) 5.6×10^{-5}
f) 2.304×10^{-1}

Q2:

a) 192
b) 30.51
c) 42900000
d) 0.0103
e) 0.00000008862
f) 0.000009512

Quiz questions (page 14)

Q3: c) 1.3

Q4: d) 39 J

Q5: c) 40.9 J

Appendix C: Data analysis

Quiz (page 28)

Q1: a) $\pm$ 0.25 V

Q2: d) $\pm$ 0.2 s

Q3: c) I and II only

Q4: a) Temperature rise

Q5: b) $\pm$ 0.1 m s^{-2}

Q6: d) 10%

Q7: c) Mass = 5.0 $\pm$ 0.2 kg

Q8: d) $\pm$ 3.0 x 10^7 m s^{-1}

Q9: b) 14.48 N $\pm$ 0.08N

Q10: d) I and II only

Accurate, precise, both or neither? (page 36)

Expected answer

a) Is neither precise nor accurate.

b) Is precise and accurate.

c) Is precise but inaccurate.

Example (page 37)

Expected answer

These numbers are clearly precise enough for us to believe that if we measure it again we would get (1.04 $\pm$ 0.01)m. The small uncertainty shows how precise our results are. These meaurements may be precise but are not accurate as the mean value is 0.04m larger than a metre stick should be.